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# Strings from quivers, membranes from moose

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Abstract: We consider $\mathcal{N} = 2$ moose/quiver gauge theories corresponding to $N_1$ D3-branes at a $\mathbb{C}^2/\mathbb{Z}_{N_2}$ singularity in the "large moose" limit where $N_1$ and $N_2$ are scaled to infinity together. In the dual holographic description, this scaling gives rise to a maximally supersymmetric pp-wave background with a *compact* light-cone direction. We identify the gauge theory operators that describe the Discrete Light-Cone Quantization (DLCQ) of the string in this background. For each discrete light-cone momentum and winding sector there is a separate ground state and Fock space. The large moose/quiver diagram provides a useful graphical representation of the string and its excitations. This representation has a natural explanation in a T-dual language. The dual theory is a non-relativistic type-IIA string wound around the T-dual direction, and bound by a quadratic newtonian potential. We end with some comments on D-string/D-particle states, a possible lift to M-theory and the relation to deconstruction.

Keywords: M-Theory, Penrose limit and pp-wave background.



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## Contents





## 1. Introduction

It is believed that gauge theories encode the dynamics of strings. Attempts to physically realize this idea led to the large-N expansion of gauge theory [1]. In this context, string theories have indeed been seen to emerge from gauge theories, most notably in Matrix Theory [2, 3] and in the AdS/CFT correspondence [4].

Matrix theory gives, in principle, a precise definition for non-perturbative M-theory using large-N supersymmetric quantum mechanics. Toroidal compactifications of M-theory are then described by large-N gauge theories in various dimensions. The construction of strings in terms of gauge fields can be made rather explicit in some cases. The best example is Matrix String Theory [5, 6, 7]. Here, perturbative string states and also D-branes in type-IIA string theory can be identified in terms of operators of a two-dimensional $\mathcal{N} = 8$ supersymmetric gauge theory.

The AdS/CFT correspondence provided a very explicit map between gauge theory operators and string theory states in the supergravity limit [8, 9]. Although the proposal can be implicitly extended to the full string theory, it proves difficult to construct strings explicitly out of gauge fields in this framework. Recently, new insight was obtained by considering a special limit of the AdS background of type-IIB string theory, in which it is possible to construct string states from gauge theory operators [10]. This is achieved by taking a Penrose limit [11] of the background, which provides a maximally supersymmetric solution of type-IIB string theory [12], and relating this to a special subset of operators in $\mathcal{N} = 4$ supersymmetric gauge theory.

Penrose limits of spacetimes generally lead to pp-wave backgrounds, which are exact backgrounds for string propagation [13, 14] and are exactly solvable in the light-cone gauge [15, 16]. In light-cone quantization, it is often useful to compactify a null direction. This leads to the Discrete Light-Cone Quantization (DLCQ), which provides a convenient regulator for string theory [17]. In this picture, one has interacting strings carrying quantized units of the light-cone momentum, with the minimal momentum being carried by a "string bit". The idea of DLCQ is most natural in the context of Matrix Theory [18], where it has been argued that each sector of the DLCQ of M-theory is exactly described by a U($N$) matrix model. Although a compact null direction may look somewhat strange, it is best thought of as the limit [19] of a spacelike compactification.

Our aim is to study type-IIB string theory in a DLCQ pp-wave background. The Penrose limits that have been considered so far in the AdS/CFT context always lead to a pp-wave with a non-compact null direction. We will show that there is a novel scaling limit of a particular AdS background, which precisely gives rise to a DLCQ pp-wave. The radius of the null direction is a finite controllable parameter of this background. The corresponding gauge theory is an $\mathcal{N} = 2$ superconformal "moose" or "quiver" theory [20].

A number of fascinating aspects of the gauge theory/pp-wave correspondence will unfold as we explore this model. As in previous examples following the original ideas of ref [10], we find gauge theory operators which can be identified to the string ground state, zero-mode oscillators and excited oscillator states. However, there are remarkable differences, and a much richer structure appears due to the DLCQ nature of the string background. We find a gauge theory operator that describes a string ground state for every value of the (positive) quantized light-cone momentum $k$. We will also construct operators that describe winding states of the string over the null direction. The gauge theory operators automatically possess all the properties required of DLCQ string states.

Our construction turns out to have a tantalizing property: the operators that describe DLCQ momentum states have the structure of a "string" of operators that winds around the moose/quiver diagram. This is suggestive of a string winding state. Conversely, the gauge theory operators that describe winding of the DLCQ string look very much like momentum states. This suggests that we should consider T-dualization of the DLCQ pp-wave to find the most direct correspondence with the gauge theory. We perform the requisite T-duality and find a theory of non-relativistic strings [21, 22, 23] bound in a harmonic-oscillator potential. The strings are wound around a compact spatial direction which can be identified with the "theory space" direction of the quiver. On lifting to M-theory we find non-relativistic membranes winding around a 2-torus and bound in a potential. Both these observations suggest that the moose/quiver theory, in our scaling limit, provides a precise realization of the concept of "deconstruction" put forward in refs [24]–[27].

This paper is organized as follows. In section 2, we describe the moose/quiver gauge theory and its AdS dual in some detail, and exhibit the scaling limit that we consider. In section 3, we discuss the DLCQ pp-wave background and show how it arises in our scaling limit. We also discuss general properties of string propagation in this background. section 4 is devoted to the identification of spacetime quantum numbers with charges of gauge-theory operators. In section 5 we explicitly construct gauge theory operators to describe various string excitations in the DLCQ pp-wave. section 6 deals with T-duality of the null direction to obtain a non-relativistic string, while section 7 gives a physical interpretation of the relationship between quivers and strings. In section 8 we make some conjectural remarks about how D-strings and other non-perturbative objects may be described in our formalism, and exhibit the lift of the IIA non-relativistic string to M-theory. Finally, we relate our work to the deconstruction idea, and close with some comments.

## 2. The large moose/quiver theory and its holographic dual

It has been known for some time that one can get four-dimensional conformal field theories by placing D3-branes at orbifolds [20]. These theories admit an AdS dual where the compact 5-manifold is an orbifold of $\mathbf{S}^5$ [28]. The specific case we will consider here is obtained by starting with $N_1$ D3-branes transverse to the 6-dimensional space $\mathbb{C}^3/\mathbb{Z}_{N_2}$. In the covering space there are $N_1 N_2$ D3-branes, which define a "parent" $\mathcal{N}=4$ super-Yang-Mills theory, of which the orbifold theory is a projection.

The orbifold group $\mathbb{Z}_{N_2}$ acts on $\mathbb{C}^3$ by:

$$(z_1, z_2, z_3) \to (z_1, \omega z_2, \omega^{-1} z_3)\,, \qquad \omega = e^{2\pi i/N_2}\,. \tag{2.1}$$

The theory on the brane world-volume is a $\mathcal{N}=2$ superconformal field theory in four dimensions, with the R-symmetry group $\mathrm{U}(1)_R \times \mathrm{SU}(2)_R$. The gauge group is

$$\mathrm{SU}(N_1)^{(1)} \times \mathrm{SU}(N_1)^{(2)} \cdots \mathrm{SU}(N_1)^{(N_2)}\,. \tag{2.2}$$

The fields in the vector multiplet for each factor of the gauge group are denoted ($A_{\mu I}$, $\Phi_I$, $\psi_{aI}$) with $I$ labelling the gauge group, $I = 1, \ldots, N_2$, and $a = 1, 2$. In addition, there are hypermultiplets $(A_I, B_I, \chi_{aI})$, where the $A_I$ are bi-fundamentals in the $(N_1, \bar{N}_1)$ of $\mathrm{SU}(N_1)^{(I)} \times \mathrm{SU}(N_1)^{(I+1)}$ and the $B_I$ are bi-fundamentals in the complex conjugate representation $(\bar{N}_1, N_1)$. The matter content of the gauge theory can be succintly summarised in the form of a quiver/moose diagram, see figure 1. Here we use the $\mathcal{N}=1$ language to describe the $\mathcal{N}=2$ gauge theory, which will be of use later. The fields $\Phi_I$, $A_I$ and $B_I$ can be identified with the $z_1$, $z_2$ and $z_3$ directions of the $\mathbb{C}^3$.

The holographic dual of the quiver theory in question is type-IIB string theory on $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_{N_2}$. The action of $\mathbb{Z}_{N_2}$ is obtained by thinking of the 5-sphere as embedded in $\mathbb{R}^6 \sim \mathbb{C}^3$ where the action is as prescribed in (2.1). This leaves a fixed circle along an equator of $\mathbf{S}^5$. The $\mathrm{AdS}_5$ space has a radius given by

$$R^2 = \sqrt{4\pi g_s^B \alpha'^2 N_1 N_2}\,, \qquad (2.3)$$

where $g_s^B$ is the type-IIB string coupling. There are also $N_1 N_2$ units of 5-form flux through the $\mathrm{AdS}_5$.

Because of the orbifold action, the volume of $\mathbf{S}^5/\mathbb{Z}_{N_2}$ is reduced by a factor $N_2$ compared to that of the covering space $\mathbf{S}^5$, with the latter having the same radius as that of $\mathrm{AdS}_5$ given in eq. (2.3). Similarly, there are $N_1$ units of 5-form flux through the $\mathbf{S}^5/\mathbb{Z}_{N_2}$ factor, which descend from $N_1 N_2$ units of flux in the covering space.

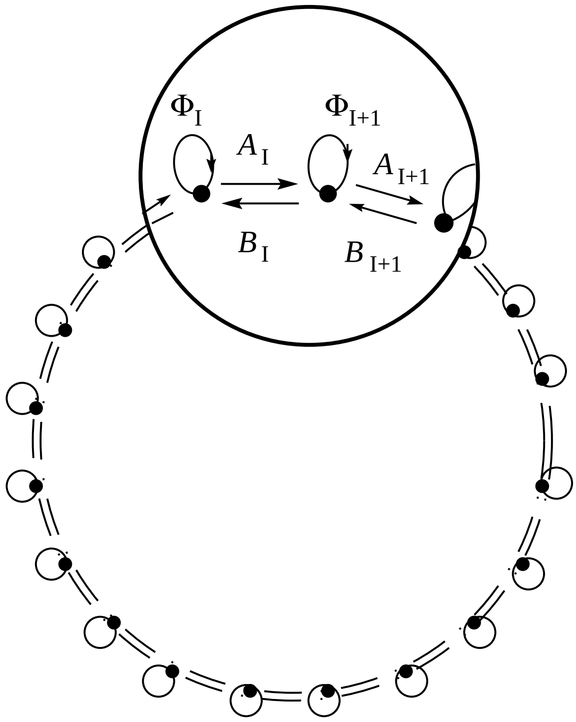


**Figure 1:** In the large $N_2$ limit the moose/quiver diagram of the gauge theory contains an large number of nodes. The two lines connecting each pair of nodes correspond to the bifundamental fields $A_I$ and $B_I$ and the line going back to the same node represents the adjoint scalars $\Phi_I$.

It is also worth noting that the coupling constant in each of the gauge group factors is given in terms of the type-IIB coupling constant as

$$(g_{YM})_I^2 = 4\pi g_s^B N_2\,. \qquad (2.4)$$

This means that the 't Hooft coupling relevant for each factor is

$$\lambda = (g_{YM})_I^2 N_1 = 4\pi g_s^B N_1 N_2\,. \qquad (2.5)$$

This is the same as the 't Hooft coupling on the original $N_1 N_2$ D3-branes before orbifolding, for which the Yang-Mills coupling was equal to $4\pi g_s^B$.

In the following, we will consider a scaling limit when both $N_1$ and $N_2$ become large, with the ratio $N_1/N_2$ held fixed. In this limit the 't Hooft coupling $\lambda$ diverges. As argued in ref [10] in the context of $\mathrm{SU}(N)$ super-Yang-Mills theory with $\mathcal{N} = 4$ supersymmetry, the relevant quantity that needs to be kept finite, is $g_s^B N/J^2$ where $J$ is a U(1) charge and the relevant states have very large $J$. In our case, we will see that the role of $J$ is played by $N_2$, while $N$ is replaced by $N_1 N_2$. So the quantity that should be kept finite for us is $g_s^B N_1/N_2$. This is achieved precisely by scaling $N_1$ and $N_2$ to infinity and keeping $g_s^B$ small but finite.

## 3. The DLCQ pp-wave

In this section we start from the holographic description of the quiver gauge theory under consideration. We will take a limit of the dual spacetime, which is known as the Penrose limit [11] (cf. [29], for generalization to supergravity). The essential idea is to consider a null geodesic and look at the spacetime in the neighbourhood of the geodesic. It was first demonstrated by Penrose that this is a sensible limit to consider in any geometry and the result is always what is known as a plane-parallel wave or pp-wave for short.

### 3.1 The Penrose limit of $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_{N_2}$

As mentioned above, to obtain the Penrose limit of any gravitational background, one has to focus on a light-like geodesic. In the particular case of $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_{N_2}$, we choose a null geodesic which is based at the origin of AdS$_5$ and carries some angular momentum along the compact directions. Because of the singular nature of the compact manifold, the result depends on whether one takes this trajectory to lie along the singular locus or not. The former choice results in a pp-wave background that has the $\mathbb{Z}_{N_2}$ ALE singularity as part of its transverse space [30]-[36]. The latter choice, which was briefly discussed in ref. [33], results instead in the maximally supersymmetric pp-wave background. In this paper we focus on the latter case as it is relevant for the scaling limit in which we are interested.

Let us write the metric of $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_{N_2}$ as:

$$\begin{aligned} ds^2 \, = \, R^2 \Bigg[ & -\cosh^2 \rho \, dt^2 + d\rho^2 + \sinh^2 \rho \, d\Omega_3^2 + \\ & + d\alpha^2 + \sin^2 \alpha \, d\theta^2 + \cos^2 \alpha \, \left( d\gamma^2 + \cos^2 \gamma \, d\chi^2 + \sin^2 \gamma \, d\phi^2 \right) \Bigg] \, , \end{aligned} \tag{3.1}$$

where the first line is the AdS$_5$ metric in global coordinates, while the remaining terms describe the metric for an $\mathbf{S}^5$ embedded in a 6-dimensional space containing a $\mathbb{Z}_{N_2}$ ALE singularity. The relationship with the complex $z_1$ coordinates and the angles is:

$$z_1 = R \sin \alpha e^{i\theta} \, , \qquad z_2 = R \cos \alpha \cos \gamma e^{i\chi} \, , \qquad z_3 = R \cos \alpha \sin \gamma e^{i\phi} \, . \tag{3.2}$$

In this parametrization the orbifold is obtained by demanding that the angles $\chi$ and $\phi$ are periodic modulo $2\pi$ but in addition have a combined periodicity under

$$\chi \to \chi + \frac{2\pi}{N_2} \, , \qquad \phi \to \phi - \frac{2\pi}{N_2} \, . \tag{3.3}$$

Note that with this choice there are no explicit factors of $N_2$ occurring in the metric.

To take the pp-wave limit, we now define new coordinates $r$, $w$, $y$ by

$$r = \rho R \, , \qquad w = \alpha R \, , \qquad y = \gamma R \, . \tag{3.4}$$

and introduce the light-cone coordinates

$$x^+ = \frac{1}{2} \left( t + \chi \right) , x^- \, = \, \frac{R^2}{2} \left( t - \chi \right) . \tag{3.5}$$

Making the substitutions the metric (3.1) becomes

$$ds^2 = R^2\Bigg[ -\cosh^2\frac{r}{R}\left(dx^+ + \frac{1}{R^2}dx^-\right)^2 + \frac{dr^2}{R^2} + \sinh^2\frac{r}{R}d\Omega_3^2 + \frac{dw^2}{R^2} + \tag{3.6}$$
$$+\sin^2\frac{w}{R}d\theta^2 + \cos^2\frac{w}{R}\left(\frac{dy^2}{R^2} + \cos^2\frac{y}{R}\left(dx^+ - \frac{1}{R^2}dx^-\right)^2 + \sin^2\frac{y}{R}d\phi^2\right)\Bigg].$$

In the limit $R \to \infty$ the metric reduces to [33]

$$ds^2 = -4dx^+dx^- - \left(r^2 + w^2 + y^2\right)dx^{+2} + dr^2 + r^2 d\Omega_3^2 + dw^2 + w^2 d\theta^2 + dy^2 + y^2 d\phi^2\,. \tag{3.7}$$

This is just the universal pp-wave background which has been found in many other cases. It can be written in the standard form

$$ds^2 = -4dx^+dx^- - \sum_{i=1}^{8}\left(x^i\right)^2 dx^{+2} + \sum_{i=1}^{8} dx^{i\,2}\,, \tag{3.8}$$

where we introduced the eight transversal coordinates $x^i$. There is also a Ramond-Ramond flux in the geometry (3.8):

$$F_{+1234} = F_{+5678} = \text{const.} \tag{3.9}$$

Although our model gives rise to the standard pp-wave metric in the Penrose limit, there is actually an important difference: the lightlike direction $x^-$ is *compact*. From eq. (3.3), the $2\pi/N_2$ periodicity of the angle $\chi$ translates into the following periodicity condition on the light-cone coordinates

$$\begin{aligned} x^+ &\to x^+ + \frac{\pi}{N_2} \\ x^- &\to x^- + \frac{\pi R^2}{N_2}\,. \end{aligned} \tag{3.10}$$

This combined shift in $x^+$ and $x^-$ has to be accompanied by a simultaneous shift in $\phi \to \phi - 2\pi/N_2$. Under a scaling $N_1 \sim N_2$, we have $R^2 \sim N_2$. Now if $N_2 \to \infty$ we see that $x^-$ is periodic in the limit, with period:[1]

$$\frac{\pi R^2}{N_2} = 2\pi R_-\,, \quad R_- = \left(\pi g_s^B \frac{N_1}{N_2}\right)^{\frac{1}{2}} \alpha'\,. \tag{3.11}$$

After taking the limit, there are no longer accompanying shifts in $x^+$ and $\phi$. So the periodic direction has become pure lightlike. As a consequence the corresponding light-cone momentum $2p^+$ is quantized in units of $1/R_-$. The way the compact null direction arises from a limit of a spacelike circle is exactly as discussed in [19] in the context of Matrix theory.

In other words, the Penrose limit of $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_{N_2}$ with $N_2 \to \infty$ leads to a Discrete Light-Cone Quantization (DLCQ) of the string on a pp-wave background, in which the null direction $x^-$ is periodic. We now turn to a discussion of string propagation in such a spacetime.

[1] Note that $x^-$ as defined has dimensions of (length)$^2$.

### 3.2 String propagation in DLCQ pp-wave

String propagation in pp-wave backgrounds is a subject which has been explored extensively in the literature. A very interesting fact about these backgrounds is that they are solutions to the world-sheet beta function equations (in covariant quantization) to all orders [13, 14], thereby being exact string backgrounds. The pp-wave backgrounds admit a covariantly constant null Killing vector, implying that one can always choose to quantize the sigma model in light-cone gauge [15]. One can extend the analysis to strings on Ramond-Ramond backgrounds following [16].

Let us begin by writing down the sigma model action for the pp-wave geometry (3.8):

$$S = -\frac{1}{4\pi\alpha'}\int d\sigma d\tau \left(-4\partial_a x^+\partial^a x^- + \partial_a x^i \partial^a x^i - \sum_{i=1}^{8}\left(x^i\right)^2 \partial_a x^+ \partial^a x^+\right). \tag{3.12}$$

The worldsheet equations of motion resulting from this are

$$\begin{aligned} \partial_a\partial^a x^+ &= 0 \\ \partial_a\partial^a x^i - x^i\left(\partial_a x^+\partial^a x^+\right) &= 0\,. \end{aligned} \tag{3.13}$$

In the light-cone gauge the first equation is solved by $x^+ = \tau$, where at the same time one takes $\sigma$ to range from 0 to $2\pi\alpha' p^+$. In addition, in order to maintain reparametrization invariance, one requires that the world-sheet stress tensor vanishes. This can be expressed as:

$$\begin{aligned} 2\partial_\tau x^- &= \frac{1}{2}\left(\left(\partial_\tau x^i\right)^2 + \left(\partial_\sigma x^i\right)^2 + \left(x^i\right)^2\right) \\ 2\partial_\sigma x^- &= \partial_\tau x^i \partial_\sigma x^i\,. \end{aligned} \tag{3.14}$$

From these equations we can easily derive the light-cone hamiltonian for the string and also derive the momentum constraints. Note that the presence of the mass term $\sum_{i=1}^{8}(x^i)^2$, for the transverse scalars inhibits the separation of the scalars into left and right movers. So far we have ignored the world-sheet fermions in the discussion. Just like in the standard pp-wave background they are given by massive Dirac fermions in the light-cone gauge. They get their mass from the Ramond-Ramond background flux, supporting the pp-wave geometry (3.9).

The solution to the world-sheet theory proceeds in the usual way by normal mode expansion and introduction of oscillators. In particular, mode expansion of the transverse coordinates $x^i$ is

$$\begin{aligned} x^i(\sigma,\tau) &= \sum_{n=-\infty}^{\infty} a_n^i \frac{1}{\sqrt{\omega_n}} e^{i\frac{n}{p^+\alpha'}\sigma + i\omega_n\tau} + \text{h.c.} \\ \omega_n &= \sqrt{1 + \frac{n^2}{(p^+)^2\alpha'^2}}\,. \end{aligned} \tag{3.15}$$

One very important fact is that with the compact $x^+$ direction, the light-cone momentum $p^+$ is quantized. This means that we have a positive integer $k$ labelling our states, with

$$2p^+ = \frac{k}{R_-}\,. \tag{3.16}$$

As is well-known, the theory then splits into sectors, labelled by a discrete number parametrising the light-cone momentum.

The hamiltonian and total momentum of the world-sheet theory are given as

$$H = \sum_{n=-\infty}^{\infty} N_n \sqrt{1 + \frac{4n^2 R_-^2}{k^2 \alpha'^2}}$$
$$P = \sum_{n=-\infty}^{\infty} n N_n \,, \tag{3.17}$$

where $N_n$ is the total occupation number of the Fourier mode labeled by $n$. These are related to the usual Virasoro generators as $L_0 = \frac{1}{2}(H_{l.c} + P)$ and $\bar{L}_0 = \frac{1}{2}(H_{l.c} - P)$.

Since we are dealing with strings, we should also expect to find states with non-zero winding number $m$. These arise as follows. If we expand $x^-$ in a mode expansion we will get oscillators $a_n^-$, which can be solved in terms of the transverse scalars. In addition we can have a zero-mode piece $m\sigma R_-$, the usual winding term, since $x^-$ is compact. Note that $m$ can take any integral value, positive or negative. So we can label our string states as $\mid k, m\rangle$. These will be our string ground states in the sector with DLCQ momentum $k$ and winding number $m$. We can further act on these states by the transverse oscillators to build other string states. A general string state can therefore be denoted as

$$\prod_{j=1}^{M} a_{n_j}^{\dagger} \mid k, m\rangle \,, \tag{3.18}$$

where for convenience of notation we dropped the superscript $i$, denoting the particular transverse coordinate on the oscillators.

The world-sheet reparametrization invariance gives a constraint on the action of the oscillators for the states written in eq. (3.18). This arises from the second equation in (3.14) and implies

$$\sum_{j=1}^{M} n_j = km \,. \tag{3.19}$$

In the next section, we will turn our attention to the construction of operators in the gauge theory which will describe the string states.

## 4. Identification of charges and light-cone momenta

As has been noted in other cases [30, 31, 32], a maximally supersymmetric pp-wave limit implies that the gauge theory has a sector which is maximally supersymmetric. A key ingredient in identifying this sector of the moose/quiver gauge theory is the interpretation of the light-cone momenta $p^+$ and $p^-$ in terms of the global symmetries of the gauge theory.

The $R$-symmetry group of the $\mathcal{N} = 2$ quiver gauge theory is $\mathrm{SU}(2)_R \times \mathrm{U}(1)_R$. Now, recall that $\Phi$ is associated with the $z_1$ direction of the $\mathbb{C}^3$, while $A$ and $B$ are related to $z_2$ and $z_3$ respectively. The $\mathrm{U}(1)_R$ factor corresponds to the transformation $z_1 \to e^{i\xi} z_1$, and therefore acts as phase rotations on the $\Phi$ scalars. The $A$ and $B$ fields have charge zero under this $\mathrm{U}(1)_R$. The $\mathrm{SU}(2)_R$ symmetry acts on the $A$ and $B$ fields and their complex conjugates. Indeed these fields form a $\mathcal{N}=2$ hypermultiplet, which is known to have a quaternionic structure. In fact, $(A,\bar{B})$ as well as $(\bar{A},B)$ form doublets under $\mathrm{SU}(2)_R$. Hence, one of the generators of this $\mathrm{SU}(2)_R$ acts on $A$ and $B$ as phase rotations. We will denote this generator by $J'$. In addition there is a $\mathrm{U}(1)$ symmetry that is not an $R$-symmetry. This is the $\mathrm{U}(1)$ symmetry that rotates $A$ and $B$ in opposite directions, and corresponds to $z_2 \to e^{i\xi} z_2$, $z_3 \to e^{-i\xi} z_3$. But, because of the orbifold identification in the $(z_2, z_3)$ directions (2.1), having $\xi = 2\pi/N_2$ brings us back to the same point. Its generator $J$ together with the $\mathrm{U}(1) \subset \mathrm{SU}(2)_R$ generator $J'$ will appear in the definition of the light-cone momentum.

In terms of the coordinates on the $\mathbf{S}^5/\mathbb{Z}_{N_2}$ the generators $J$ and $J'$ are given by

$$J = -\frac{i}{2N_2}\left(\partial_\chi - \partial_\phi\right), \qquad J' = -\frac{i}{2}\left(\partial_\chi + \partial_\phi\right). \tag{4.1}$$

This leads to the following identifications for the light-cone momenta

$$\begin{aligned} 2p^- &= i(\partial_t + \partial_\chi) \;=\; \Delta - N_2 J - J' \\ 2p^+ &= i\frac{(\partial_t - \partial_\chi)}{R^2} \;=\; \frac{\Delta + N_2 J + J'}{R^2}. \end{aligned} \tag{4.2}$$

In our conventions, the light-cone hamiltonian is $H = 2p^-$.

As explained in ref. [10], to relate gauge theory operators to string states, we need to look for operators that have both $p^-$ and $p^+$ finite. Since $R \to \infty$, this means $\Delta$ and $N_2 J + J'$ must both be large, while their difference remains finite. Physical gauge invariant operators should have half-integral values for $J$ and $J'$. This implies that $N_2 J$ automatically becomes large when $N_2 \to \infty$ even when $J$ is kept fixed. We will see that $J'$ also grows like $N_2$. The scaling dimension $\Delta$ also becomes large automatically, because we have the BPS bound $\Delta \geq N_2 J + J'$. In fact, if we keep $\Delta - N_2 J - J'$ fixed, then both quantities precisely grow in the right way for the pp-wave limit, provided we take $N_1$ and $N_2$ simultaneously to infinity with the ratio $N_1/N_2$ fixed. In this double scaling limit we have $R^2 \sim \sqrt{N_1 N_2} \sim N_2$, and so indeed $p^+$ and $p^-$ stay both finite. We would like to emphasize again that, in contrast with the other pp-wave backgrounds considered so far, there is no need in our case to send $J$ to infinity. In fact, in order to reproduce the DLCQ spectrum we have to keep $J$ finite, since it will give the discrete value of the light-cone momentum. We have $J = \frac{1}{2}k$.

We now discuss the $(\Delta, J, J')$ eigenvalues of the various local operators in the gauge theory, and construct the string ground state and oscillators in terms of these. Because of $\mathcal{N}=2$ supersymmetry, all the fundamental bosonic fields have exact conformal dimension 1, the same as their free field value, while the fermions similarly have dimension 3/2.

The charges are obtained as follows. The $A_I$ and $B_I$ fields that make up the hypermultiplets have fractional charge under $J$. The reason is that $e^{4\pi i J}$ precisely generates the orbifold transformation $z_2 \to \omega z_2$, $z_3 \to \omega^{-1} z_3$. The $A$ and $B$ fields transform accordingly, and hence have charge $1/2N_2$ and $-1/2N_2$ respectively. The operator $J'$ generates a $\mathrm{U}(1)$ symmetry contained in the $\mathrm{SU}(2)_R$ factor of the R-symmetry group $\mathrm{U}(1)_R \times \mathrm{SU}(2)_R$. Under

| | $\Delta$ | $N_2 J$ | $J'$ | $H$ |
|---|---|---|---|---|
| $A_I$ | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 |
| $B_I$ | 1 | $-\frac{1}{2}$ | $\frac{1}{2}$ | 1 |
| $\Phi_I$ | 1 | 0 | 0 | 1 |
| $\chi_{AI}$ | $\frac{3}{2}$ | $\frac{1}{2}$ | 0 | 1 |
| $\chi_{BI}$ | $\frac{3}{2}$ | $-\frac{1}{2}$ | 0 | 2 |
| $\psi_{\Phi I}$ | $\frac{3}{2}$ | 0 | $-\frac{1}{2}$ | 2 |
| $\psi_I$ | $\frac{3}{2}$ | 0 | $-\frac{1}{2}$ | 2 |

**Table 1:** Dimensions and charges for chiral fields and gauginos

| | $\Delta$ | $N_2 J$ | $J'$ | $H$ |
|---|---|---|---|---|
| $\overline{A}_I$ | 1 | $-\frac{1}{2}$ | $-\frac{1}{2}$ | 2 |
| $\overline{B}_I$ | 1 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 1 |
| $\overline{\Phi}_I$ | 1 | 0 | 0 | 1 |
| $\overline{\chi}_{AI}$ | $\frac{3}{2}$ | $-\frac{1}{2}$ | 0 | 2 |
| $\overline{\chi}_{BI}$ | $\frac{3}{2}$ | $\frac{1}{2}$ | 0 | 1 |
| $\overline{\psi}_{\Phi I}$ | $\frac{3}{2}$ | 0 | $\frac{1}{2}$ | 1 |
| $\overline{\psi}_I$ | $\frac{3}{2}$ | 0 | $\frac{1}{2}$ | 1 |

**Table 2:** Dimensions and charges for complex conjugate fields

this $\mathrm{U}(1) \subset \mathrm{SU}(2)_R$, the fields $\Phi_I$ are neutral since they correspond to translations of the original $N_1 N_2$ D3-branes along the transverse $\mathbb{R}^2$ that is unaffected by the orbifold group. On the other hand, the scalars $A_I, B_I$ in the hypermultiplets both have charge $\frac{1}{2}$ under $J'$. Complex conjugation and supersymmetry give us the remaining charge assignments, for the the fermions and all the conjugate fields.

The dimension and charge assignments, along with the $H = 2p^-$ values, are summarized in tables 1 and 2. In table 1, $A_I$, $B_I$ refer to the scalar components of the $\mathcal{N} = 1$ chiral superfields that form the $\mathcal{N} = 2$ hypermultiplets. $\chi_{A_I}$, $\chi_{B_I}$ are their fermionic partners. $\Phi_I$ are the complex scalars in the vector multiplet, while $\psi_{\Phi I}$ are their fermionic partners. Finally, $\psi_I$ are the gauginos in the theory. Table 2 lists the complex conjugate fields.

## 5. String states from gauge theory operators

The duality between type-IIB string theory and the quiver gauge theory implies that all string states must have corresponding states in the gauge theory. As emphasized in ref. [10] one can use radial quantization to map the gauge theory states on $\mathbf{S}^3 \times \mathbb{R}$ to local operators. Therefore, string states are holographically dual to operators in the gauge theory. In this section we will identify these operators in the large moose/quiver theory and discuss how they match the string spectrum in the DLCQ pp-wave background. The construction will be similar in spirit to the BMN-operators[2] [10] in the $\mathcal{N} = 4$ case, but there will be important differences in the details. One of the main differences is that in our case we find a set of operators that matches the DLCQ string spectrum in the pp-wave background, which carry *discrete* light-cone momenta label by an integer $k$ as well as discrete winding numbers $m$.

To explain the basic construction and to simplify the notation we will first discuss the states in the $k = 1$ DLCQ sector. These states correspond to a single "bit" of string carrying the smallest possible momentum, namely $2p_+ = 1/R_-$. After that we explain how to put these string bits together to form strings with arbitrary light-cone momentum.

[2]The acronym BMN refers to the authors of ref. [10], viz., Berenstein, Maldacena and Nastase.

### 5.1 DLCQ ground states and zero mode oscillators

The simplest state in the string theory is the $|k = 1, m = 0\rangle$ DLCQ ground state. According to our identification of $p^+$ this should be a state with $\Delta = N_2 J + J' = N_2$. Since this state has $H = 0$, it is clear that it has to be constructed out of the $A_I$ alone. The $A_I$ fields are bi-fundamental with respect to the pair of gauge groups $\mathrm{SU}(N_1)^{(I)} \times \mathrm{SU}(N_1)^{(I+1)}$, hence the simplest gauge-invariant operator that can be made out of them should contain all $N_2$ $A_I$ fields precisely once. We thus arrive at the identification

$$|k = 1, m = 0\rangle = \frac{1}{\sqrt{\mathcal{N}}} \mathrm{Tr}\,(A_1 A_2 \cdots A_{N_2}) \tag{5.1}$$

with $\mathcal{N} = N_1^{N_2}$. This operator has $H = 0$ and $\Delta = N_2$, and is illustrated in figure 2, where it appears as a string of $A_I's$ wrapping once around the moose diagram. The normalization of the operator can be determined by looking at the two-point function in the free field limit. Basically, each of the $A_I$ has to contract with the corresponding object and we get a factor of $N_1$ from each gauge group trace.

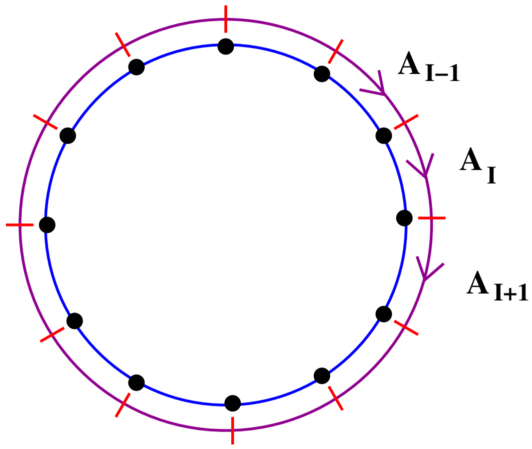


**Figure 2:** The $k = 1$ ground state, $|k = 1, m = 0\rangle \sim \mathrm{Tr}\,(A_1 A_2 \cdots A_{N_2})$.

The states with arbitrary light-cone momentum are obtained by literally stringing together the $k = 1$ string bits. In particular, the ground states in the sector with $k$ units of momentum is described by

$$|k, m = 0\rangle = \frac{1}{\sqrt{\mathcal{N}^k}} \mathrm{Tr}\left(\underbrace{A_1 A_2 \cdots A_{N_2} A_1 A_2 \cdots A_{N_2} \ldots\ldots\ldots A_1 A_2 \cdots A_{N_2}}_{k \text{ times}}\right). \tag{5.2}$$

Thus we have a single gauge-invariant operator with $H = 0$ and $\Delta = N_2 k$ for each value of $k$. One easily checks that it has the right properties to describe the DLCQ ground state with momentum $2p^+ = k/R_-$. The $k = 2$ operator is pictorially represented in figure 3 as a string of $A_I$'s that wraps twice around the moose.

String oscillator modes are obtained by inserting the various fields with $H = 1$ in the appropriate locations in the string of A-fields. We will first discuss the oscillator states for the case $k = 1$. In this case the notation is simpler, and furthermore, it will more clearly exhibit the differences between the operators in the quiver theory and the BMN-operators of the $\mathcal{N} = 4$ Yang-Mills theory.

We now come to the first excited states in the $k = 1$ sector with light-cone hamiltonian $H = 1$. For the string in the pp-wave background, these states are obtained by acting once with a single zero mode oscillator on the ground states. There are eight bosonic zero mode oscillators, corresponding to the transversal coordinates $x^i$, so we expect to find eight bosonic states with $H = 1$. In the gauge theory these states are obtained by inserting appropriate combinations of the $\Phi$ and $B$ fields and covariant derivatives into the trace of the string of A-fields.

From the dimensions and charges of the bosonic operators listed in the table, it is clear that we can admit precisely one insertion of a covariant derivative, a $\Phi_I$, $B_I$, or their complex conjugates, since they all have $H = 1$. The matrix nature of these fields (adjoint or bi-fundamental) constrains what gauge-invariant operators can be written down. For example, the fields $\Phi_I$ are in the adjoint of $SU(N_1)^{(I)}$ and therefore must be inserted between $A_{I-1}$ and $A_I$ in the string of operators. The same is true for $\bar{\Phi}_I$ and the covariant derivatives $D_i^{(I)}$. The field $B_I$, however, is a bifundamental and requires an extra insertion of $A_I$, while $\overline{B}_I$ can only be inserted in the place of $A_I$.

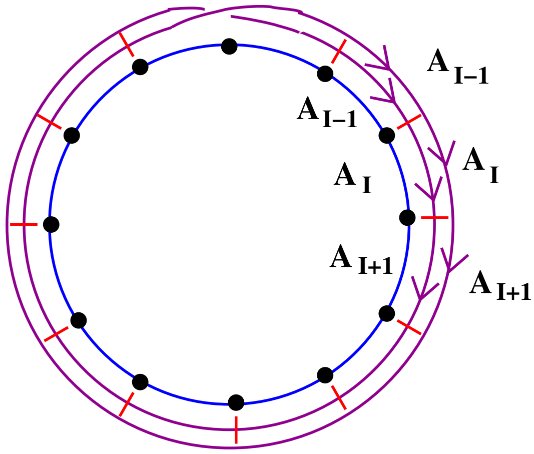


**Figure 3:** The $k = 2$ ground state, $|k = 2, m = 0\rangle \sim \mathrm{Tr}\,(A_1 A_2 \cdots A_{N_2} A_1 A_2 \cdots A_{N_2})$.

In this way we get the following set of operators with $H = 1$. First, for the $\Phi$ fields we have

$$a^{\dagger}_{\Phi,0}|k=1, m=0\rangle = \frac{1}{\sqrt{N_1 N_2 \mathcal{N}}} \sum_{I=1}^{N_2} \mathrm{Tr}\,(A_1 A_2 \cdots A_{I-1} \Phi_I A_I \cdots A_{N_2}) \tag{5.3}$$

$$a^{\dagger}_{\bar{\Phi},0}|k=1, m=0\rangle = \frac{1}{\sqrt{N_1 N_2 \mathcal{N}}} \sum_{I=1}^{N_2} \mathrm{Tr}\,(A_1 A_2 \cdots A_{I-1} \bar{\Phi}_I A_I \cdots A_{N_2})\,. \tag{5.4}$$

The expressions for the covariant derivatives $D_i^{(I)}$ are similar, and will not be written explicitly. The states containing $B$ fields are:

$$a^{\dagger}_{B,0}|k=1, m=0\rangle = \frac{1}{\sqrt{N_1^2 N_2 \mathcal{N}}} \sum_{I=1}^{N_2} \mathrm{Tr}\,(A_1 A_2 \cdots A_I B_I A_I \cdots A_{N_2}) \tag{5.5}$$

$$a^{\dagger}_{\overline{B},0}|k=1, m=0\rangle = \frac{1}{\sqrt{N_2 \mathcal{N}}} \sum_{I=1}^{N_2} \mathrm{Tr}\,(A_1 A_2 \cdots A_{I-1} \overline{B}_I A_{I+1} \cdots A_{N_2})\,. \tag{5.6}$$

The sum over the position of the insertions is necessary to ensure that we are describing the zero mode fluctuations. This differs from the BMN operators $\mathrm{Tr}\,(Z^J \Phi)$ for which the sum was automatically implemented by the cyclicity of the trace. Thus we have found the eight expected bosonic states at $H = 1$. The operators involving insertions of $\Phi, \bar{\Phi}$ are represented in figure 4, and those representing insertions of $B, \bar{B}$ in figure 5.

In addition there are eight fermion states. They are obtained in a similar way by inserting the fermionic partners of these fields. The complex $A$, $B$ and $\Phi$ scalars each have a Weyl doublet fermionic partner, that together make up 12 real fermionic fields. In

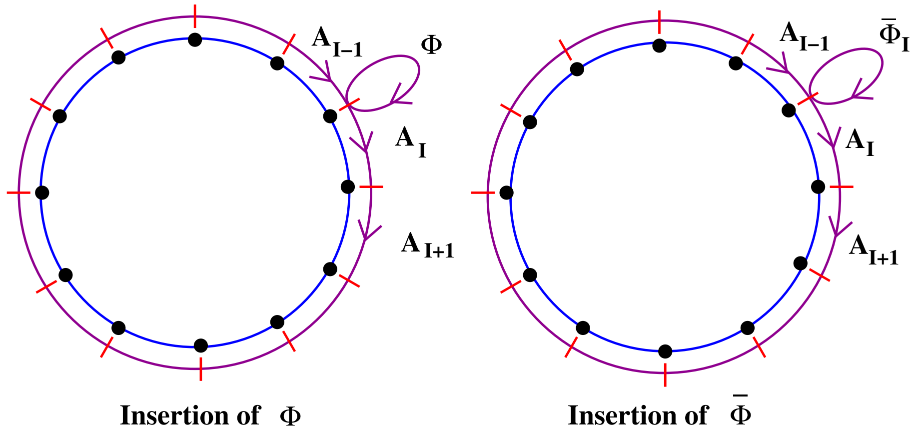


**Figure 4:** The building blocks for the states $a^\dagger_{\Phi,0}|k=1,m=0\rangle$ and $a^\dagger_{\bar{\Phi},0}|k=1,m=0\rangle$.

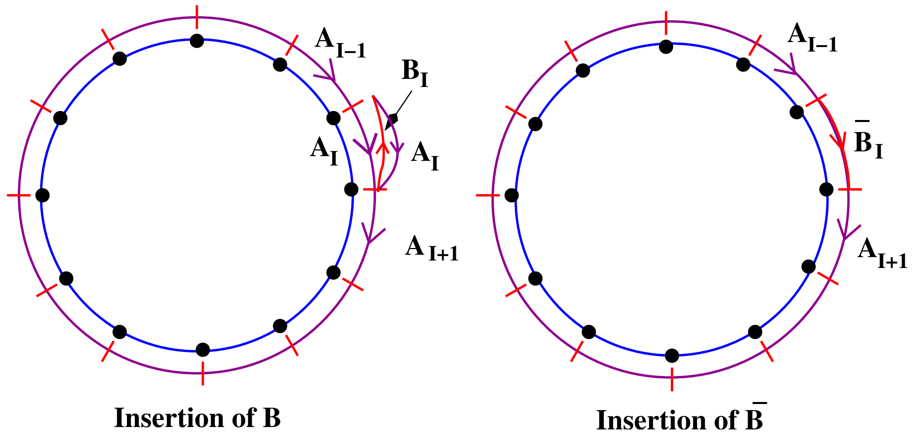


**Figure 5:** The building blocks for the states $a^\dagger_{B,0}|k=1,m=0\rangle$, $a^\dagger_{\bar{B},0}|k=1,m=0\rangle$.

addition we have two gaugino fields, making up 16 fermions in the theory. From the table, we see that precisely half of these have $H=1$, and these are the ones that are used to construct the eight fermionic oscillators of the string. Four of these are associated to $\chi_A$ and $\bar{\chi}_B$, which are superpartners of the $A$ fields under $\mathcal{N}=2$ supersymmetry.

### 5.2 Winding states and non-zero mode oscillators

An important difference between our large moose/quiver theory and the $\mathcal{N}=4$ Yang-Mills theory is that the A-fields carry labels, and have to appear in a particular order inside the trace. This means that a field that is inserted somewhere in the string of $A$'s will have a definite position associated with it which can not be changed using cyclicity of the trace. This fact will be important in what follows because it allows us to identify the states with non-zero winding along the light-cone. To keep the notation simple we consider insertions of only $\Phi$ fields. The extension of our discussion to other fields will be straightforward. We also drop the normalization factors from here on.

JHEP05(2002)023

Similarly as in ref. [10] we can construct operators in which the sum over the locations of the fields includes phase factors. The simplest states of this kind are the single oscillator states with winding number $m$

$$a^\dagger_{\Phi,m}|k=1,m\rangle = \sum_{I=1}^{N_2} \mathrm{Tr}\,(A_1A_2\cdots A_{I-1}\Phi_I A_I \cdots A_{N_2})\omega^{mI}\,, \tag{5.7}$$

where $\omega = e^{\frac{2\pi i}{N_2}}$. Note that, unlike the cases previously studied, these operators do not vanish due to the cyclicity of the trace. This is just as well, because we want to have these states in the DLCQ spectrum of the string. More generally we can construct operators with multiple insertions of $\Phi$-fields corresponding to states with more oscillators.

$$\prod_{i=1}^{M} a^\dagger_{\Phi,n_i}|k=1,m\rangle = \sum_{l_M>\cdots>l_2>l_1} \mathrm{Tr}\,(A_1\cdots A_{l_1-1}\Phi_{l_1}A_{l_1}\cdots A_{l_i-1}\Phi_{l_i}A_{l_i}\cdots A_{N_2})\,\omega^{\sum n_i l_i}\,, \tag{5.8}$$

where the winding number $m$ is defined as the sum of the mode numbers $n_i$:

$$m \equiv \sum_i n_i\,. \tag{5.9}$$

The operators (5.8) represent the most general perturbative string states in the $k=1$ sector with one unit of light-cone momentum.

### 5.3 Matrix notation

Before we turn to the operators with general $k$, we would like to rewrite our result for $k=1$ in a concise notation. For this purpose, it will be convenient to introduce $(N_1N_2)\times(N_1N_2)$ matrices to represent the $(\Phi_I, A_I, B_I)$ as follows:

$$\mathbf{A} \equiv \begin{pmatrix} 0 & A_1 & 0 & \cdots & 0 \\ 0 & 0 & A_2 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & A_{N_2-1} \\ A_{N_2} & 0 & 0 & \cdots & 0 \end{pmatrix} \qquad \mathbf{B} \equiv \begin{pmatrix} 0 & 0 & \cdots & 0 & B_{N_2} \\ B_1 & 0 & \cdots & 0 & 0 \\ 0 & B_2 & \cdots & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & B_{N_2-1} & 0 \end{pmatrix} \tag{5.10}$$

and

$$\mathbf{\Phi} \equiv \begin{pmatrix} \Phi_1 & 0 & \cdots & 0 \\ 0 & \Phi_2 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \Phi_{N_2} \end{pmatrix}, \tag{5.11}$$

where the blocks $A_I$, $B_I$, $\Phi_I$ are $N_1\times N_1$ matrices.

The $k=1$ ground state is simply $\mathbf{Tr}\,(\mathbf{A}^{N_2})$, where $\mathbf{Tr}$ represents the trace over the larger $N_1N_2$-dimensional space. The eight bosonic states with $H=1$ may up to normalization be written as

$$\mathbf{Tr}\left(\mathbf{\Phi}\mathbf{A}^{N_2}\right), \quad \mathbf{Tr}\left(\bar{\mathbf{\Phi}}\mathbf{A}^{N_2}\right), \quad \mathbf{Tr}\left(\mathbf{B}\mathbf{A}^{N_2+1}\right), \quad \mathbf{Tr}\left(\bar{\mathbf{B}}\mathbf{A}^{N_2-1}\right), \quad \mathbf{Tr}\left(D_i\mathbf{A}^{N_2}\right). \tag{5.12}$$

In this notation the expression of our operators are very similar to the BMN-operators $\mathrm{Tr}\,(Z^J\Phi)$ etc. This is not a coincidence, because our operators can be obtained from theirs via an orbifold projection.

In the sector with zero winding number we can write the states with many oscillators as:

$$\prod_{i=1}^{M} a^{\dagger}_{\Phi,n_i}|k=1,m=0\rangle = \sum_{l_M>\cdots>l_2>l_1}^{N_2} \mathbf{Tr}\,\left(\mathbf{A}^{l_1}\mathbf{\Phi}\mathbf{A}^{l_2-l_1}\mathbf{\Phi}\cdots\mathbf{\Phi}\mathbf{A}^{N_2-l_M}\right)\omega^{\sum n_i l_i}\,. \tag{5.13}$$

It follows from cyclicity of the trace that the r.h.s.vanishes unless $\sum_i n_i = 0 \pmod{N_2}$. Cyclicity is implemented by the shift $l_i \to l_i+1$, which causes the expression to pick up a phase $\omega^{\sum n_i}$. This phase must be equal to 1 and hence the total mode number is set to zero modulo $N_2$. This all seems nice and well, but we appear to have lost our winding states! This is not surprising because we have written our states in a notation that is inherited from the parent $\mathcal{N}=4$ theory. The winding states are not present in the parent theory, but originate as twisted sectors in the orbifold.

To describe the states with non-zero winding we introduce the clock matrix:

$$\mathbf{V} \equiv \omega \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \omega & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \omega^{N_2-1} \end{pmatrix}. \tag{5.14}$$

where the additional phase is introduced for convenience. This is actually an $N_1N_2\times N_1N_2$ matrix made up of $N_1\times N_1$ blocks proportional to the identity. It obeys

$$\mathbf{AV} = \omega\mathbf{VA} \tag{5.15}$$

and commutes with $\Phi$. To obtain the operators in the sector of winding number $m$, we insert an explicit factor of $\mathbf{V}^m$ in the trace, say, at the end. As a result a cyclic permutation of all the other operators in the trace produces an extra phase $\omega^m$. Hence the argument that first gave a zero total mode number now gives indeed that $\sum_i n_i = m$. In fact, we can use the clock matrix $V$ to rewrite the states with many oscillators in a way that does not require any explicit phases to be inserted. Namely, we associate to each oscillator a matrix-valued operator given by:

$$a^{\dagger}_{\Phi,n} \leftrightarrow \mathbf{\Phi}_n \equiv \mathbf{\Phi}\mathbf{V}^n\,. \tag{5.16}$$

The matrix on the r.h.s. is diagonal and has entries $\Phi_I\omega^{nI}$. Then the general oscillator state for momentum $k=1$ is simply given by:

$$\prod_{i=1}^{M} a^{\dagger}_{\Phi,n_i}|k=1,m=\sum_i n_i\rangle = \sum_{l_M>\cdots>l_2>l_1}^{N_2} \mathbf{Tr}\,\left(\mathbf{A}^{l_1}\mathbf{\Phi}_{n_1}\mathbf{A}^{l_2-l_1}\mathbf{\Phi}_{n_2}\cdots\mathbf{\Phi}_{n_M}\mathbf{A}^{N_2-l_M}\right), \tag{5.17}$$

where no additional phases are inserted. The winding number of this state is the total "mode number" of the $\mathbf{\Phi}_n$, which is just the number of $\mathbf{V}$ matrices inside the trace.

For future purpose we also introduce the shift matrix

$$\mathbf{U} \equiv \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{pmatrix}. \tag{5.18}$$

The clock and shift matrices obey the familiar relation

$$\mathbf{UV} = \omega \mathbf{VU}\,.$$

We can for example use the shift matrix $\mathbf{U}$ to shift all $A_I$ fields in the $\mathbf{A}$ matrix by the map $\mathbf{A} \to \mathbf{U}^{-1}\mathbf{AU}$. Applying this map to the $\mathbf{A}$ and $\mathbf{\Phi}$ matrices in the operators (5.17) gives back the same operator, but multiplied by a phase $\omega^m$ where $m$ is the winding number. This observation will be useful when we consider the generalization to arbitrary light-cone momentum $k$.

### 5.4 String states at arbitrary light-cone momentum

To describe the operators for general $k$ we make use of a similar matrix notation as just described for the $k = 1$ states. To this end we introduce even bigger $\mathbf{A}, \mathbf{B}$ and $\mathbf{\Phi}$ matrices of size $kN_1N_2$, where for example

$$\mathbf{A} \equiv \begin{pmatrix} 0 & A_1 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & A_2 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & & & \ddots & & & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & A_{N_2-1} & 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & A_{N_2} & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & A_1 & \cdots & 0 \\ \vdots & & & \vdots & & & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & A_{N_2-1} \\ A_{N_2} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \end{pmatrix} \tag{5.19}$$

where the string of entries above the diagonal is repeated $k$ times before terminating in the lower left corner. We continue to use the previous notation to label these larger matrices.

The ground state in the momentum $2p^+ = k/R_-$ sector can simply be written as $\mathbf{Tr}\,(\mathbf{A}^{N_2k})$. Then, in a similar way as for $k = 1$, we can introduce $N_2k \times N_2k$ shift and clock matrices $\mathbf{U}$ and $\mathbf{V}$, where $\mathbf{U}$ again has unit entries just off the diagonal and in the left lower corner, while

$$\mathbf{V} = \mathrm{diag}\left(\omega^{\frac{1}{k}}, \omega^{\frac{2}{k}}, \ldots, \omega^{\frac{(N_2k-1)}{k}}\right). \tag{5.20}$$

These matrices now satisfy:

$$\mathbf{UV} = \omega^{\frac{1}{k}} \mathbf{VU}\,. \tag{5.21}$$

The general oscillator states can be written as in eq. (5.17) in terms of the new $\mathbf{A}, \mathbf{\Phi}$ and $\mathbf{V}$. We get

$$\prod_{i=1}^{M} a^\dagger_{\Phi,n_i}|k,m\rangle = \sum_{l_M > \cdots > l_2 > l_1}^{N_2k} \mathbf{Tr}\left(\mathbf{A}^{l_1}\mathbf{\Phi}_{n_1}\mathbf{A}^{l_2-l_1}\mathbf{\Phi}_{n_2}\cdots\mathbf{\Phi}_{n_M}\mathbf{A}^{N_2k-l_M}\right). \tag{5.22}$$

In this case, the sum of all oscillator mode numbers is equal to

$$\sum_i n_i = km\,. \tag{5.23}$$

To see that the l.h.s.is a multiple of $k$, we note that the $\mathbf{A}$ and $\mathbf{\Phi}$ matrices are invariant under shifting all entries over $N_2$ places. Thus, we have

$$\mathbf{U}^{-N_2}\mathbf{A}\mathbf{U}^{N_2} = \mathbf{A}\,, \qquad \mathbf{U}^{-N_2}\mathbf{\Phi}\mathbf{U}^{N_2} = \mathbf{\Phi}\,. \tag{5.24}$$

We now apply the first relation to replace all the $\mathbf{A}$ matrices inside the trace. By conjugating the $\mathbf{U}$ matrices through the $\mathbf{V}$ and $\mathbf{\Phi}$ matrices one picks up a phase $\omega^{N_2 \sum_i \frac{n_i}{k}}$. In effect we have not changed anything: we simply shifted all $A_I$ fields by $N_2$ steps, which gives back the same operator. Therefore, the operator vanishes unless the phase is equal to one. Thus, one concludes that the sum of all mode numbers is indeed a multiple of $k$.

In most of the above discussion, we have focussed on the oscillators constructed out of $\mathbf{\Phi}$. However, it is straightforward to see that similar expressions hold for the remaining oscillators, with $\mathbf{\Phi}$ replaced by $\bar{\mathbf{\Phi}}$, $\mathbf{AB}$, $\mathbf{A}^{-1}\bar{\mathbf{B}}$ and $D_i$, or one of the fermionic fields. This completes our construction of perturbative string states for the pp-wave background with a compact light-cone direction. The entire spectrum of the string in the DLCQ pp-wave is thus reproduced by this set of operators of the $\mathcal{N} = 2$ gauge theory. They represent a sector of the gauge theory with maximal supersymmetry.

The picture of gauge theory operators winding around a circular moose diagram, which has a circumference of order $N_2$ and is therefore very large, is rather suggestive. Even though these operators describe momentum modes in type-IIB string theory, they appear to build up a string winding over a spatial dimension, Similarly, the gauge theory operators describing winding modes in type-IIB string theory have the appearance of momentum states. This strongly suggests that T-duality is involved. In the next section we will carry out the relevant T-duality explicitly and exhibit the relation between the large moose diagram and the T-dual string.

This is also somewhat related to the idea of "deconstruction" [24]–[27] in which a spatial dimension is created by taking a suitable limit of a moose/quiver theory. However, as we will explain in section 9, our limit differs from the one in ref. [26].

## 6. T-duality of the DLCQ pp-wave background

The periodicity of the $x^-$ direction is a remnant of the combined $2\pi/N_2$ periodicity in the angles $\chi$ and $\phi$, exhibited in eq. (3.3). Before taking the limit $N_1, N_2 \to \infty$ the periodic direction was space-like. Hence, one can perform a Buscher-type T-duality along this periodic direction.

To perform the T-duality, we go back to the original metric in eq. (3.1) and write down only the terms in the $t$ and $\chi$ directions explicitly:

$$ds^2 = R^2\left[-\cosh^2\rho dt^2 + \cos^2\alpha\cos^2\gamma d\chi^2\right] + ds^2_{\text{transverse}}\,. \tag{6.1}$$

Next we make the substitutions defined in eq. (3.4), and express $\chi$ in terms of $t$ and $x^-$ as in eq. (3.5). As a result, the metric becomes:

$$ds^2 = R^2\left(\cos^2\frac{w}{R}\cos^2\frac{y}{R} - \cosh^2\frac{r}{R}\right)dt^2 - 4\cos^2\frac{w}{R}\cos^2\frac{y}{R}\,dt\,dx^- + \\ + \frac{4}{R^2}\cos^2\frac{w}{R}\cos^2\frac{y}{R}(dx^-)^2 + h_{ij}\left(\frac{x^i}{R}\right)dx^i dx^j\,, \tag{6.2}$$

where we parametrize the transverse metric by $h_{ij}$. We know that $h_{ij}\left(x^i/R\right) \to \delta_{ij}$ as $R \to \infty$.

We see that, before taking the pp-wave limit, $x^-$ is spacelike: there is a small $g_{--}$ in the metric of order $1/R^2$. The metric of the transverse space, which we have not written down explicitly here, is the usual flat metric with no factors of $R^2$ in front. For the transverse space we can consistently ignore corrections incest $1/R^2$, since those directions will be unaffected by T-duality.

We now perform a T-duality along the spacelike direction $x^-$, following the usual duality rules. Note that this is a different T-duality than the one considered in [37]. After this, we denote the T-dual coordinate by $2x^9$, to end up with the metric:

$$ds^2 = -R^2\cosh^2\frac{r}{R}\,dt^2 + \frac{R^2}{\cos^2\frac{w}{R}\cos^2\frac{y}{R}}\left(dx^9\right)^2 + h_{ij}\,dx^i dx^j\,, \tag{6.3}$$

along with a B-field and dilaton:

$$B_{t\,9} = -R^2\,, \qquad g_s^A = \frac{\sqrt{\alpha'}R}{R_-\cos\frac{w}{R}\cos\frac{y}{R}}\,g_s^B\,. \tag{6.4}$$

Note that $x^9$ now has period $2\pi\alpha'/R_-$, with $R_-$ as given in eq. (3.11).

Now let us take the limit $R \to \infty$. We see that some components of the metric, and the $B$-field and string coupling, become infinite in this limit. However, it turns out that string propagation on this background is finite. The reason is that the $B$-field is a critical electric field, and cancels the leading divergent piece in the string world-sheet action $\sqrt{-\det(\gamma)} + B$ where $\gamma_{ab} = g_{ab} + \partial_a x^i \partial_b x^i$ is the induced metric in static gauge. Here $g_{ab}$ denotes the spacetime metric in the $(t, x^9)$ plane. Momentarily ignoring the derivative terms, we find:

$$\begin{aligned}\sqrt{-\det(g)} + B &= R^2\frac{\cosh\frac{r}{R}}{\cos\frac{w}{R}\cos\frac{y}{R}} - R^2 \\ &= \frac{1}{2}\left(r^2 + w^2 + y^2\right) + \mathcal{O}\left(\frac{1}{R^2}\right) \\ &= \frac{1}{2}\sum_{i=1}^{8}\left(x^i\right)^2 + \mathcal{O}\left(\frac{1}{R^2}\right).\end{aligned} \tag{6.5}$$

If we put back the derivative terms, the above calculation will instead give:

$$\sqrt{-\det(\gamma)} + B = \frac{1}{2}\sum_{i=1}^{8}\left[\partial_a x^i \partial^a x^i + \left(x^i\right)^2\right] + \mathcal{O}\left(\frac{1}{R^2}\right). \tag{6.6}$$

This is just the non-relativistic string propagating in a background with a newtonian potential of harmonic-oscillator type. Indeed, the leading dependence on $R$ in eqs. (6.3), (6.4) above is identical to that which appears in the Non-Commutative Open String (NCOS) [38, 39] and Non-Relativistic Closed String (NRCS) theories [21, 22, 23]. Our model is therefore an NRCS theory. However, because of the pp-wave metric that we started with, it inherits a harmonic oscillator potential in which the light closed strings are bound.

To see that the effective coupling constant of this non-relativistic theory is finite, we recall that the coupling for the NR closed strings winding in the direction of the critical electric field is the same as for the NC open strings. The latter is well-known to be [40]:

$$g_o^2 = g_s \sqrt{\frac{\det(g+B)}{\det g}}. \tag{6.7}$$

Inserting the appropriate values from eqs.(6.3), (6.4) above and taking $R \to \infty$, we see that for our model,

$$g_o^2 = \frac{g_s^B}{R_-} \sqrt{\alpha' \sum_{i=1}^{8} (x^i)^2}\,. \tag{6.8}$$

So the effective coupling for the type-IIA NR closed string is independent of $R$ in the limit, but it varies in the transverse directions. Because of the potential, low-lying states of the string are localized in the region around $x^i \sim 0$, and on dimensional grounds one expects that the square root in the above equation is replaced by $\alpha'$ for such states.

There is also a 5-form Ramond-Ramond background on the type-IIB side that must be T-dualized. To start with, we had:

$$F^{(5)} = R^4\, dt \wedge (dV)_4 + R^4\, d\chi \wedge (dV')_4 \tag{6.9}$$

where $dV$, $dV'$ are the 4-forms on the $(\rho, \Omega_3)$ and $(\alpha, \theta, \gamma, \phi)$ spaces respectively. We use eq. (3.4) and the definition of $x^-$ in eq. (3.5), and take the limit $R \to \infty$. Then the above expression becomes:

$$F^{(5)} = dt \wedge (dr)_4 + dt \wedge (dw)_2 \wedge (dy)_2 - \frac{2}{R^2} dx^- \wedge (dw)_2 \wedge (dy)_2\,. \tag{6.10}$$

Next, performing a T-duality, we find a 4-form and a 6-form field strength on the type-IIA side:

$$\begin{aligned} F^{(6)} &= 2dt \wedge (dr)_4 \wedge dx^9\,, \\ F^{(4)} &= -\frac{2}{R^2} (dw)_2 \wedge (dy)_2\,. \end{aligned} \tag{6.11}$$

Thus we have an electric 6-form and a magnetic 4-form field strength. As one can check, the two are dual to each other. These RR fields will give masses to the worldsheet fermions of the string, which are necessary for maximal supersymmetry, just as the 5-forms do on the type-IIB side.

In section 8 we discuss the lift of this background to M-theory, on which it gives rise to an analogous non-relativistic membrane wrapped on a 2-torus.



JHEP05(2002)023

## 7. Large quiver as non-relativistic string

We have argued that the quiver gauge theory with gauge group $\mathrm{SU}(N_1)^{N_2}$ with large $N_1, N_2$ is dual to type-IIB string theory on the pp-wave background with a compact lightlike direction $x^-$. We have also shown that one can perform a T-duality over the lightlike direction, thereby going to a type-IIA description in terms of a non-relativistic closed string theory (NRCS). In the following, we present a number of observations which address the physical meaning of this correspondence. We will argue that the large moose/quiver theory deconstructs the non-relativistic string, and eventually M-theory, in a rather precise way.

We have described how ground states and oscillators of type-IIB string theory on a DLCQ pp-wave background can be constructed from gauge theory operators. A key ingredient was the construction of a string ground state for every (positive integer) DLCQ momentum $k$. This state was associated to the trace of a "string" of bi-fundamental $A$ fields wrapping $k$ times around the "theory space" defined by the quiver diagram. Although in the type-IIB language it is a momentum state, in the quiver theory it is reminiscent of a winding state of some other string theory. Similarly, the states describing the winding of IIB strings around the DLCQ direction, obtained using the $\mathbf{V}$-matrices, look very much like momentum states of a string theory.

We now see that this other string theory is precisely the type-IIA NRCS theory! It is known that in NRCS theory, along the direction of the critical electric field there are light states which are closed-string winding modes. However, the winding can take place only in one orientation around the circle. The physical reason is simple [21]. Consider the Non-Commutative Open String (NCOS) theories obtained by turning on critical electric $B$-fields over a D-brane. In these theories, we know that open strings are very light when they align one way along the electric field, and very heavy when they are lined up the other way. Now if the direction along the electric field is compact, an open string can align along it and stretch most of the way around. When its end points come close to each other, and because the open-string coupling is finite, this string can close up and become a closed string wound on the compact direction. As a result, NCOS theories on compact directions include light winding strings. If now we remove the open-string sector by taking away the original D-brane, we will be left with the closed strings wound in one direction as the light states. This is the NRCS theory.

Evidently these NRCS winding modes are just the T-dual states of the DLCQ momentum modes. Just as DLCQ momenta are always positive and never negative (the latter become infinitely heavy), the NRCS winding modes are light for one orientation and very heavy for the other. Thus winding states of the non-relativistic closed string are identified with the gauge theory operators wrapping our large moose/quiver.

In fact, the quiver deconstructs this string in a very precise way. The "string" of A-fields winding around the quiver in one way is the light NR closed string. A string of $\bar{A}$-fields, which wrap the quiver in the other sense, would be identified with the very heavy closed string that is wrongly aligned with the electric field, so it is not in the spectrum of light states. Single wrapping of $A_I$, corresponding to $k = 1$, is identified with a "string bit" in DLCQ language, while the $A_I$'s themselves are the latticized components of this string bit. The vanishing $p^-$ value of the $A$ fields leads to the vanishing energy of the string bit which is aligned with the electric field. And the **V** operators in the quiver theory, which created winding states of the DLCQ string, are the momenta of the NR closed string. Indeed, we can insert **V** raised to any positive or negative integer power, just like the allowed momenta of an NRCS. The quiver theory in the limit we consider is a non-relativistic string with a quadratic potential. The observations of [21, 22, 23] made in the context of flat space NRCS theory and its relation to DLCQ strings are not modified in the pp-wave background.

## 8. Beyond perturbation theory

So far we have analyzed the large moose theory at the perturbative level. In this section we will discuss some ideas on how to describe non-perturbative aspects of our theory. In particular, we propose an identification of D-string states in terms of operators in the gauge theory. We also discuss the lift of the type-IIA solution to M-theory.

### 8.1 D-string states

In addition to the fundamental string oscillator and winding states that we have discussed, we also expect to find states in the string theory that represent D-strings winding around the DLCQ direction. These would be S-dual, in the framework of type-IIB string theory, to the F-string winding states. In fact, using various elements of the $\mathrm{SL}(2,\mathbb{Z})$ S-duality group, one can construct $(p,q)$ winding strings where $p$ and $q$ are relatively prime.

The moose/quiver gauge theory that we started with is also believed to have a large S-duality group which contains $\mathrm{SL}(2,\mathbb{Z})$ as a subgroup [43]. The perturbative operators that we have considered so far are constructed out of electric variables that are weakly coupled when the Yang-Mills coupling is small. But in principle for each operator that we have constructed there must exist corresponding operators that are expressed in magnetic or even in dyonic variables. These different operators are then related by $\mathrm{SL}(2,\mathbb{Z})$ tranformations. The ground states that we considered can easily be seen to be invariant under the electromagnetic $\mathrm{SL}(2,\mathbb{Z})$ transformations. They represent supersymmetric graviton states that carry only space-time momentum. Also the states that are obtained by acting with zero mode oscillators will be invariant: they correspond to gravitons whose transversal movements are described by excited harmonic oscillator states.

The non-zero mode oscillator states and the states with non-zero winding are true F-string states, and will transform non-trivially under $\mathrm{SL}(2,\mathbb{Z})$ and are mapped on excited $(p,q)$ string states with non-zero winding. It is useful to think about these states from a T-dual perspective. After T-duality the discrete light-cone momentum $k$ becomes string winding, while string winding $m$ is mapped on to the discrete momentum along the string. D-string winding becomes identified with D-particle number. In this dual language the extra states that we are looking for are thus described by strings bound to D-particles.

Can we introduce D-particles on the quiver/moose diagram? This is a hard question, because it involves non-perturbative issues in the gauge theory, and might require the introduction of the dual magnetic gauge field. However, there appears to be a natural proposal for these D-string/particle states which only involves pure electric variables. To give some motivation for the proposal, we note that the perturbative string states represent "electric" states associated to the confining phase of the theory. This suggests that the "magnetic" states must be obtained from a dual Higgs phase. Suppose the A-fields get a vacuum expectation value. $\mathbb{Z}_{N_2}$ invariance implies that they all get the same VEV. Since the theory is conformal invariant, we can put these VEV's equal to one. Hence we have $\langle \mathbf{A} \rangle = \mathbf{U}$. In the Higgs phase, it is very natural to construct operators using the shift matrix $\mathbf{U}$ in the same way as we did for the $\mathbf{V}$-matrix. Specifically, in the $k = 1$ DLCQ sector we can define oscillators

$$a^\dagger_{\Phi,m,n} \leftrightarrow \mathbf{\Phi}_{m,n} \equiv \mathbf{\Phi}\mathbf{U}^m\mathbf{V}^n \,, \tag{8.1}$$

insert them into the trace, and sum over all locations. The F- and D-string winding numbers may then be defined as the sum of the $n$ and $m$ mode numbers respectively. The states with $n = 0$ will be pure D-strings, while those with $(n, m) = l(p, q)$ with $(p, q)$ both nonzero and relatively prime will correspond to $l$ $(p, q)$ strings. Clearly this reproduces the operators we had before, and extends them in an almost manifest $\mathrm{SL}(2, \mathbb{Z})$ invariant manner. There is one subtlety however. The number of $\mathbf{A}$ fields that are inside the trace must be reduced by the number of D-string windings, because otherwise the trace would vanish. The $U$ matrices play the role of 'place holders' and create 'holes' in the string of A-fields. This is just because the fields are replaced by their vacuum expectation values. These holes are the locations of the D-particles on the string.

This can be generalized to general $k$ values by extending $\mathbf{U}$, $\mathbf{V}$, $\mathbf{A}$, $\mathbf{\Phi}$ to be $(kN_1N_2) \times (kN_1N_2)$ matrices, and requiring that the resulting states be invariant. The D-string/particle number is then defined in terms of the phase that is picked up when we replace all $\mathbf{A}$ fields by $\mathbf{V}\mathbf{A}\mathbf{V}^{-1}$ and similarly for the $\mathbf{\Phi}$'s. This phase will be an $N_2$'th root of unity, and from $\mathbf{U}\mathbf{V} = \mathbf{V}\mathbf{U}\omega^{1/k}$ we get

$$\omega^{\sum \frac{m_i}{k}} \equiv \omega^{\tilde{m}} \,, \tag{8.2}$$

where $\omega = e^{\frac{2\pi i}{N_2}}$ and $\tilde{m}$ is the D-string winding number.

We should perhaps emphasize that the proposed description of the D-string/particle states came from a heuristic argument and was not derived in a precise way. However, we are pretty sure that there must be states that carry these quantum number, basically because of the $\mathrm{SL}(2, \mathbb{Z})$ symmetry of the underlying gauge theory. At finite coupling the gauge theory contains perturbative as well as non-perturbative states, and therefore it should also give a non-perturbative of the IIB string in the DLCQ background description, or the type-IIA string in the dual background. If this is indeed true than we should conclude that the large quiver/moose theory also has a dual M-theory description in which the F- and D-string states combine to form the states of a membrane that wraps the $x^9$ as well as the $x^{10}$ M-theory circle. In the next section we will describe the M-theory background that is dual to the DLCQ pp-wave and in which the membrane is living.



JHEP05(2002)023

### 8.2 Lift to M-theory

The type-IIA background T-dual to the DLCQ pp-wave, described in eqs. (6.3), (6.4) can be lifted to M-theory in the standard way at finite $R$. It will be convenient to rewrite these formulae as follows. Define:

$$H(w,y) = \frac{1}{g_s^B}\frac{R_-}{\sqrt{\alpha'}R}\cos\frac{w}{R}\cos\frac{y}{R}\,. \tag{8.3}$$

In addition, we rescale $x^9$ so that it has periodicity $2\pi\sqrt{\alpha'}$. The type-IIA background is now given by:

$$\begin{aligned} ds^2 &= -R^2\cosh^2\frac{r}{R}\,dt^2 + \frac{1}{\left(g_s^B\right)^2 H^2}\left(dx^9\right)^2 + h_{ij}\,dx^i dx^j\,, \\ B_{t\,9} &= -\frac{R^2}{R_-}\,, \qquad e^{\phi} = \frac{1}{H}\,. \end{aligned} \tag{8.4}$$

This solution lifts to the following M-theory background:

$$\begin{aligned} \frac{1}{(l_p)^2}ds_{11}^2 &= H^{\frac{2}{3}}\left[-R^2\cosh^2\frac{r}{R}\,dt^2 + H^{-2}\left(\frac{1}{\left(g_s^B\right)^2}\left(dx^9\right)^2 + \left(dx^{10}\right)^2\right) + h_{ij}\,dx^i dx^j\right], \\ C_{t\,9\,10} &= -\frac{R^2}{R_-}\,. \end{aligned} \tag{8.5}$$

where $x^9$ and $x^{10}$ are periodic modulo $2\pi$, and $R^2$ in this equation is just $\sqrt{4\pi g_s^B N_1 N_2}$ with no dimensional factors. From the metric, we see that the ratio of the physical radii of $x^{10}$ and $x^9$ is $g_s^B$, as expected.

Again we see that the background itself is singular as $R\to\infty$, but membrane propagation is finite and non-relativistic. Non-relativistic membranes are related to OM (open membrane) theory [41] in a similar way as non-relativistic strings are related to NCOS theories [22, 42]. We can compute the membrane action in static gauge, again ignoring derivative terms to start with:

$$\begin{aligned} \sqrt{-\det(g)} + C &= \frac{1}{g_s^B H}R\cosh\frac{r}{R} - \frac{R^2}{R_-} \\ &= \frac{1}{2R_-}\sum_{i=1}^{8}(x^i)^2 + \mathcal{O}\left(\frac{1}{R^2}\right). \end{aligned} \tag{8.6}$$

As for the string, the kinetic terms can be restored leading to a free action for the transverse scalars on the membrane world-volume. The background will also have a 7-form field strength and its dual 4-form, following from eqs. (6.11). These will give masses to the fermions of the membrane world-volume theory as required by maximal supersymmetry.

## 9. Relation to deconstruction

We have been working with the $\mathrm{SU}(N_1)^{N_2}$ $\mathcal{N}=2$ supersymmetric quiver gauge theory for large $N_1, N_2$. The same theory, for finite $N_1$ and large $N_2$, was the starting point for deconstructing $(2,0)$ superconformal field theory in six dimensions [26].



JHEP05(2002)023

The essential idea in deconstruction [24] is to start with a four-dimensional gauge theory for which the "theory space" is large. To be precise, one takes a large number of gauge groups and introduce matter in non-trivial representations of two of the gauge groups simultaneously; a prototypical example of this being the quiver gauge theory under consideration. Upon going to the Higgs branch of the theory by giving VEVs to the matter in the bifundamentals, one can at some intermediate energy scales recover approximate five-dimensional gauge dynamics. The role of the fifth dimension is played by the direction in the theory space. In the deep infra-red the theory recovers four-dimensional behaviour, since the Higgs VEV causes the large gauge group to be broken down to the diagonal subgroup. The five-dimensional gauge coupling and the lattice spacing are governed by the Higgs VEV along with other parameters of the gauge theory.

In [26], this idea was extended to supersymmetric gauge theories in four-dimensions, and the quiver $\mathcal{N}=2$ theory was the starting point for deconstruction of *two* extra dimensions. One takes a set of $N_1$ D3-branes probing a $\mathbb{C}^2/\mathbb{Z}_{N_2}$ orbifold. However, if $N_2$ happens to be large, then the resulting wedge of $\mathbb{C}^2$ is like a thin sliver. The D3-branes are moved away from the orbifold point to a distance $d$. In the dual gauge theory, this corresponds to giving a VEV to the bifundamental hypermultiplets. The D3-branes are now in a very thin cone, and being away from the tip, they essentially can be considered to be in a cylindrical geometry where the radius of the transverse circle is $d/N_2$. For large $N_2$, when this radius is vanishingly small in string units, one can T-dualize and work with D4-branes wrapping a dual circle of radius $R^A = N_2\alpha'/d$. The type-IIA coupling constant is $g_s^A = N_2\sqrt{\alpha'}/d$.

The limit considered in ref. [26] is to take $N_2$ large and $\alpha' \to 0$ with $g_s^B$ and $d/N_2\alpha'$ fixed. In this limit the type-IIA coupling blows up, so we can lift the configuration to M-theory. The M-theory circle has a radius $R_{10} = g_s^B N_2\alpha'/d$ and the 11-dimensional Planck length is $l_P^3 = (\alpha')^2 N_2 g_s^B/d$. The type-IIA four-branes have become M5-branes wrapping the $x^{10}$ circle. We are left with an M-theory background with $N_1$ M5-branes, which for $l_p \to 0$ defines the $(2,0)$ theory with parameter $N_1$.

To summarize, the basic idea in ref. [26] is to keep $N_1$ and $g_s^B$ fixed, taking $N_2$ large while staying in the Higgs branch. The states which survive in the gauge theory are of energies $\langle A\rangle/N_2$, where $\langle A\rangle$ is the Higgs expectation value.

In our approach, we go over to the AdS limit, $N_1 \to \infty$, simultaneously with the "deconstruction" limit $N_2 \to \infty$. In the AdS background, we pick a circle along the $\chi$ direction, which lies a distance $R$ away from the fixed circle of the orbifold action. This is the analog of staying away a distance $d$ from the tip of the cone in the discussion of [26]. But $d$ is not to be identified with $R$ as such, since the energy scale $R/N_2\alpha'$ is vanishingly small in the limit we consider. In addition to staying away from the fixed circle, the pp-wave limit also involves a boost along the $\chi$ direction. This introduces another factor of $R$, and therefore we need to compare the scale $d/N_2\alpha'$ of [26] with $R^2/N_2\alpha'^{3/2}$. We look at states which have energy of order the inverse of this length scale.

Recall that the radius of our compact null direction is proportional to $\sqrt{N_1/N_2}$. We now see that the conventional AdS limit and the limit of ref. [26] probe opposite ends of the DLCQ compactification moduli space, with the former corresponding to $R_- \to \infty$ and the latter corresponding to $R_- \to 0$. By a double scaling of $N_1$ and $N_2$, we have succeeded in retaining the DLCQ radius as a tunable parameter.

## 10. Discussion

We end with some final comments and speculate about using the large moose/quiver theory as a definition of M-theory.

### 10.1 Concluding comments

Our work indicates several interesting directions to explore. One such direction is to consider $\mathcal{N} = 1$ superconformal gauge theories obtained by placing $N_1$ D3-branes at a $\mathbb{C}^3/(\mathbb{Z}_{N_2} \times \mathbb{Z}_{N_3})$ orbifold singularity. The holographically dual $AdS_5 \times \mathbf{S}^5/(\mathbb{Z}_{N_2} \times \mathbb{Z}_{N_3})$ background naturally admits three different pp-wave limits. Two of these are DLCQ pp-wave backgrounds with an orbifold in the transverse space, hence they have reduced supersymmetry. The third limit, which is more interesting, gives a maximally supersymmetric pp-wave. The corresponding quiver diagram is a two-dimensional discretized torus, and the gauge theory operators wind along a diagonal of this torus, sampling all the $N_2 N_3$ points [44].

An alternative description of the $\mathcal{N} = 2$ quiver theory that we studied in this paper is provided by a type-IIA brane construction in terms of NS5 and D4-branes [43]. This construction lifts to M-theory as an M5-brane wrapped on a Riemann surface. It would be worthwhile to examine whether this approach, in our scaling limit, can be related to the non-relativistic membrane theory that we have exhibited here.

Another interesting study, similar to the deconstruction idea, deals with the matrix theory description of our quiver theory [45]. In this work, a relation is found between $\mathrm{SU}(N_1)^{N_2}$ and $\mathrm{SU}(N_2)^{N_1}$ quiver theories. This exchange of $N_1$ and $N_2$ is reminiscent of the T-duality that we perform, and might have implications for it.

Finally, we note that the DLCQ formalism is intended to facilitate the study of string interactions in a controlled fashion. It is clearly important to understand string interactions within the gauge theory/pp-wave correspondence, and the framework provided in this paper should be useful to address this issue.

### 10.2 Might 'M' mean 'Moose'?

In this paper we considered a particular limit of the $\mathcal{N} = 2$ moose/quiver theory in which not only $N_1$ and $N_2$ are taken to be larger, but at the same time one focusses on a particular sector of the theory: only operators are considered for which $\Delta$ and $J'$ grow like $N_2$ and $J$ and $\Delta - N_2 J - J'$ are kept finite. The latter conditions imply that the operators are constructed mostly out of $A$ fields. In principle one could have chosen to look at a different sector for which another combination of $\Delta$ and the charges is kept finite leading to operators that contain for example mostly $\bar{A}$, $B$ or $\bar{B}$ fields. This seems to suggest that the large moose/quiver theory may be even richer and may contain all kinds of sectors that we have not yet explored.



JHEP05(2002)023

This brings us to an important question: is one allowed to send $N_1$ and $N_2$ to infinity without taking the string coupling to zero? Or does this only makes sense provided one also restricts to an appropriate subset of operators? To see why this is an important question, let us assume that one *can* make sense of the large moose/quiver theory without any severe restrictions on the coupling or the class of operators one is considering. In that case one expects that the resulting theory is dual to type-IIB string theory on $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_{N_2}$ in limit where the radius of the $\mathrm{AdS}_5$ and the $\mathbf{S}^5$ both go to infinity, while taking also the order $N_2$ of the $\mathbb{Z}_{N_2}$ orbifold group to infinity. Without the orbifold identification one would, at least naively, expect to get the type-IIB theory in a flat background. One of the spatial direction is, however, infinitely small due to the orbifold symmetry, and hence should be replaced by a T-dual coordinate. This gives a type-IIA theory in flat space but with a coupling constant that becomes infinite. In other words, the large $N_1$ and $N_2$ limit of the moose/quiver theory, if it exists, is a serious candidate for a non-perturbative description of type-IIA string theory, that is of M-theory! So we may ask: "Might M mean moose?"



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